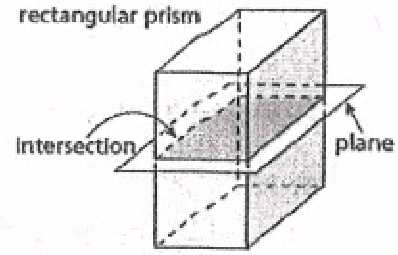
Notes 9.3 Cross Sections and Volume of Prisms

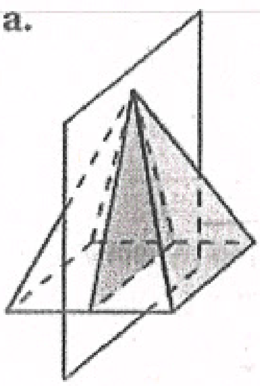
Cross Sections of Three-Dimensional Figures

Consider a plane "slicing" through a solid. The intersection of the plane and the solid is a two-dimensional shape called a **cross section.** For example, the diagram shows that the intersection of the plane and the rectangular prism is a rectangle.



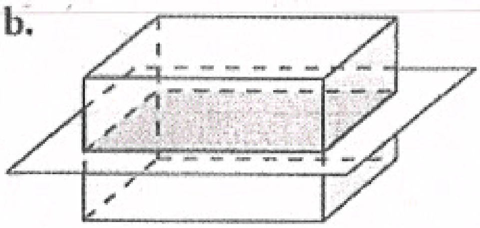
Example 1: Describe the intersection (cross section) of the plane and the solid.

a.



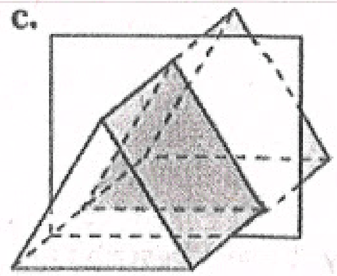
Triangle

b.



Rectangle

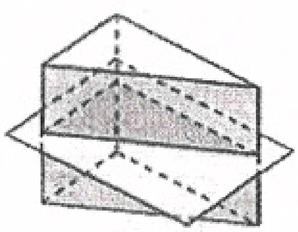
c.



Triangle

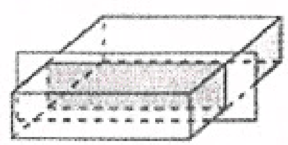
Try These: Describe the intersection (cross section) of the plane and the solid.

1.



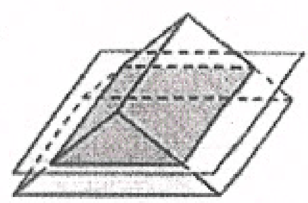
Triangle

2.



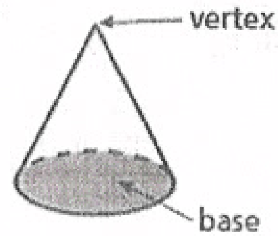
Rectangle

3.



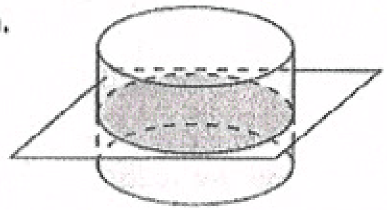
Rectangle

Example 1 shows how a plane intersects a polyhedron. Now consider the intersection of a plane and a solid having a curved surface, such as a cylinder or cone. As shown, a *cone* is a solid that has one cirular base and vertex.



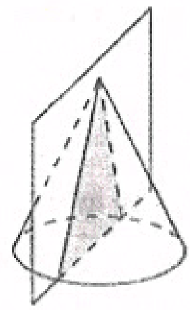
Example 2: Describe the intersection (cross section) of the plane and the solid.

a.



Circle

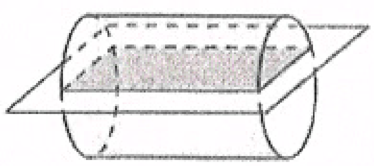
b.



Triangle

Try These: Describe the intersection (cross section) of the plane and the solid.

7.

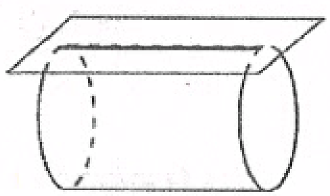


Rectangle

8.

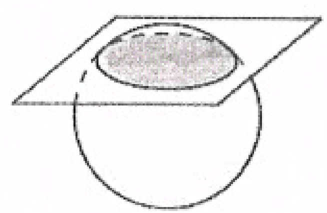
Circle

9.



Line

10.

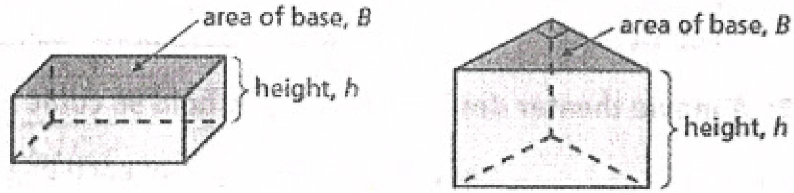


Circle

Volume of Prisms

The ***volume*** of a three-dimensional figure is a measure of the amount of space that it occupies. Volume is measured in cubic units.

Words The volume, V, of a prism is the product of the area of the base and the height of the prism.



Algebra

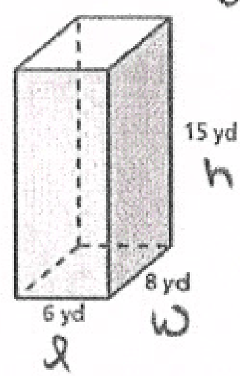
$$V = Bh$$

B: Area of base; h: Height of prism

Example 1: Find the volume of the rectangular prism.

base = rectangle

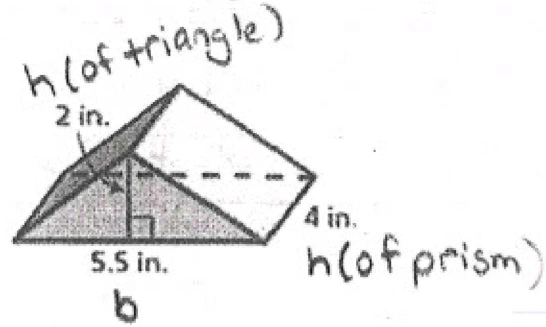
$V = Bh$	Write formula for volume.
$V = (lw)h$	Substitute lw for B.
$V = 6(8)(15)$	Substitute.
$\boxed{V = 720 \text{ yd}^3}$	Multiply.



Example 2: Find the volume of the triangular prism.

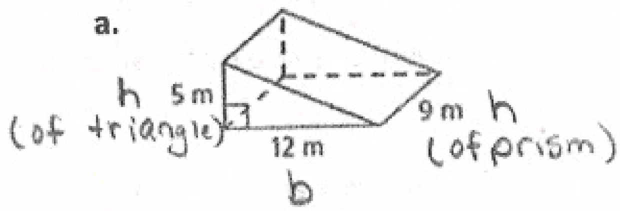
base = triangle

$V = Bh$	Write formula for volume. Base is a triangle.
$V = \left(\frac{1}{2}bh\right)h$	Substitute area of a triangle formula for B.
$V = (\frac{1}{2} \cdot 5.5 \cdot 2) \cdot 4$	Multiply.
$V = 5.5(4)$	Multiply.
$\boxed{V = 22 \text{ in}^3}$	



Try This: Find the volume of the prism.

a.



$V = Bh$

$V = (\frac{1}{2}bh)h$

$V = (\frac{1}{2} \cdot 12 \cdot 5) \cdot 9$

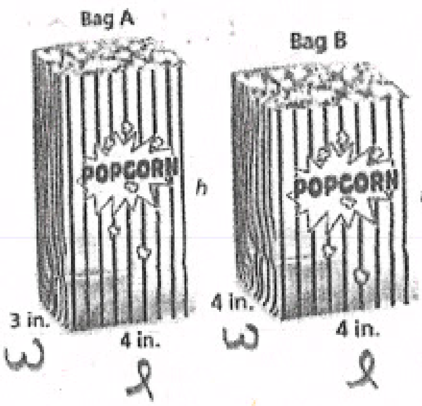
$V = 30(9)$

$\boxed{V = 270\,m^3}$

base = triangle

Example 3: A movie theater designs two bags to hold $\boxed{\text{96 cubic inches}}$ of popcorn.

Volume



base = rectangle

(a) Find the height of each bag.

Bag A	Bag B
$V = Bh$	$V = Bh$
$V = (lw)h$	$V = (lw)h$
$96 = 4 \cdot 3 \cdot h$	$96 = 4 \cdot 4 \cdot h$
$\frac{96}{12} = \frac{12h}{12}$	$\frac{96}{16} = \frac{16h}{16}$
$\boxed{8\text{ in} = h}$	$\boxed{6\text{ in} = h}$

(b) Which bag should the theater choose to reduce the amount of paper needed? (*Hint: find the surface area of each bag. Remember, there is no top).*

Bag A	Bag B
$SA = 1lw + 2lh + 2wh$	$SA = 1lw + 2lh + 2wh$
$SA = (1 \cdot 4 \cdot 3) + (2 \cdot 4 \cdot 8) + (2 \cdot 3 \cdot 8)$	$SA = (1 \cdot 4 \cdot 4) + (2 \cdot 4 \cdot 6) + (2 \cdot 4 \cdot 6)$
$SA = 12 + 64 + 48$	$SA = 16 + 48 + 48$
$\boxed{SA = 124\text{ in}^2}$	$\boxed{SA = 112\text{ in}^2}$

The theater should choose Bag B B/C $112\text{in}^2 < 124\text{in}^2$.